6.C: the ***PhilosophyIdeas*** overview of

Sources of Mathematics

The nature and foundations of mathematics are internal topics of the subject, of great interest to mathematicians, but the sources of the subject – how we come to have mathematics, and what aspects of reality produce and ground it – are mainly the concern of philosophy. The ingredients of mathematics might be taken as primitive, but usually there are three main views of the matter. Mathematics may be seen as a set of necessary and eternal truths, an inexplicable abstract realm of reality which we explore like a jungle. Or it may be seen as a highly generalised account of the structure of the physical universe. Or it may be entirely the product of human minds, which has no reality outside of our thinking, understood as a very useful tool, or as an entirely fictional realm of interlocked ideas only developed because it intrigues us. These three positions are Platonism, Empiricism and Conceptualism.

A**ncient platonism** was a strong commitment to a realm of eternal ideas. The human mind somehow has the capacity to engage with this realm, and reveal some of its truths. Since mathematics seems to be so pure and unchanging in character, our reason must directly access this realm, and studying mathematics reveals a deep level of reality. Immediate difficulties occur when we try to be precise about his proposal. If each number is an eternal idea, are smaller numbers part of larger numbers, or separate? If they are parts, then the eternal ideas are an inextricable knot too complex to grasp; if they are separate, then how do they relate to one another? A modern objection is that these ideas seem to lack causal powers which could link them to thinking minds like ours.

Modern platonism is more modest in its claims. It just claims that there is a 'third realm' which is distinct from both the world of thought and the physical world. This is the world of *abstracta*, which are understood as 'objects' which exist (or perhaps 'subsist') outside of time and space, and lack causal powers. If we assert that some sentence is true, and its subject is a proper name or a singular term, then it is assumed that the referent of the subject must exist. If I say 'the next prime after 7 is 11', I take that to be true, so these two entities must (in some way) exist. The criterion for this sort of existence is commitment to an identity statement, such as '7+4' is identical to '11'. It seems undeniable that we all treat mathematical objects as items that persist, have properties, participate in true statements, and seem to have an objective reality. It may even be that the existence of mathematics is **indispensable** to our physical sciences, and this is an 'ontological commitment' that goes with accepting scientific theories. Critics remain doubtful about entities existing without causal powers, and wonder whether we need this ghostly world that is neither mind nor matter, and exists even when no one is thinking about it. Numbers seem better understood as part of a system, than as isolated abstract objects. Even more modest is **restricted platonism**, which takes mathematical objects to exist, but only as idealised projections of thinking.

The other extreme from platonism is **empiricism**, which confines mathematics to this world. Most empiricists are **nominalists**, because they do not believe in 'abstract objects'. Extreme empiricism says that numbers and geometry are features of the physical world. A group of six pebbles has the property of 'sixness', and exhibits patterns such as 2 x 3. This account seems less plausible, though, for zero, or huge numbers, or real numbers, or higher infinities. It is not clear whether the number property of a pack of cards refers to its cards, corners, suits or pips, suggesting that our own concepts are involved. Hence empiricists usually add a psychological process of abstraction or idealisation to our experience of physical objects, introducing units, sums, straight lines and so on. Even if mathematics entirely concerns our concepts, we may have an empirical view of how those concepts originate. It may be that mathematics concerns **structures**, and only begins when our thinking about the world focuses on the pure relations. The empiricist approach places more emphasis on numbers expressed as adjectives than as nouns, since they seem to be properties of the world, rather than objects in their own right. The two modes of expression can usually be paraphrased into one another, so the priority between nouns and adjective is unclear. The main attraction of the empiricist approach is its good explanation of why mathematics **applies** well to the physical world. Platonists usually explain that by the high level of generality in mathematics, which coincides with physical generalities.

The conceptualist view is that mathematics fits the world because we have created it to do so, in the way a painted landscape fits the world. **Constructivism** is a cautious approach to mathematics, which restricts what will count as mathematics to entities and truths for which we can demonstrate a formal mode of construction. It seems plausible that concepts such as complex numbers didn't exist until they were introduced by a certain construction. **Finitism** is a compromise position, which confidently accepts finite mathematics, but treats transfinite mathematics as a rather shadowy conceptual extension of it. It appeals to our attachment to the counting numbers and simple quantifiers, but handles poorly important mathematical ideas, such as real numbers and functions.

The most drastic conceptualism is **fictionalism**, which tells us that mathematical entities have the same ontological status as characters in novels. Since nature does not contain any mathematics, the supposed applications of mathematics to nature are strictly false, although they may throw light on nature in the way that good fiction does. A very austere view of mathematics is **formalism**, which says that it is merely the manipulation of symbols, with no ontological commitments. This treats mathematics as pure syntax, with no interpretations or semantics (and no problems about gaps between the two). The **if-thenism** version says mathematics is just the consequence of axioms, with no interpretation given of the axioms. Formalism struggles to explain both infinities and the application of mathematics, and has few followers now that a rigorous account of formal truth has been developed.

An important strategy in understanding the sources of mathematics is **logicism**. The idea is to explain the ingredients of the successful axioms, which are taken to be definitions and pure analytic truths which are known a priori, and have the high level of generality we find in logic. The original logicism struggled with technical difficulties, and was obliged to offer a logic that included ontological commitments, which seemed contrary to the spirit of logic. In modern times the **neo-logicist** movement tries to prove the idea by basing arithmetic on the logic of one-to-one relations. Logicists try to avoid set theory, since that has ontological commitments which extend beyond logic.